South East Asian J. of Mathematics and Mathematical Sciences
Vol. 16, No. 1 (2020), pp. 37-50

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

A STUDY OF STEADY STATE ANALYSIS OF D/M/1 MODEL AND M/G/1 MODEL WITH MULTIPLE VACATION QUEUEING SYSTEMS

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(**Received:** *Jun. 27, 2019* **Accepted:** *Jan. 22, 2020* **Published:** *Apr. 30, 2020)*

Abstract: In this paper we have considered D/M/1 and M/G/1 queueing models with multiple vacation queueing system which consists of Type I and type II vacations. Type I vacation is taken when all the waiting customers gets served and type II vacation is availed after returning from first vacation and still finds an empty queue. In this paper, we calculate steady state for these two models when server is in working state, on type I and type II vacation. Further, we have investigated the results for the above discussed states by taking a problem of a four wheeler service centre from real world and analyze the steady state of the system. The results of analysis shows that after some time as the time increases, our system obtains the steady state i.e.; the system becomes independent of time. Steady state are shown graphically by MATLAB software of given data.

Keywords and Phrases: D/M/1 Model, M/G/1 Model, multiple vacation queueing system, steady state, waiting customers, service centre.

2010 Mathematics Subject Classification: 60K25, 68M20, 90B2.

1. Introduction

Vacation model is studied widelybecause of its various applications in diverse fields like manufacturing, communication and computer systems. Generally, we study the queueing models where server is always available even after the queue becomes empty on completion of queue. But in some situations it happens that the server becomes unavailable when all the customers of the queue gets served. It is called as a queue with vacations.Sometimes the vacation is taken voluntarily due to an empty queue and during that time the server gets busy in other secondary work. The server breakdown is one morecause for vacation in which some mechanical or may be technical problem arises. Thus, vacation is an important concept which can be used when server wants to utilize the free time in other fruitful work.The time period for which server is not there for serving the customers is called as server vacation period. It is possible that server may be unavailable for a known (fixed) period of time or may be for unknown (random) period of time. Generally, vacation system are of two types. In the former type, after the completion of vacation the server comes and finds an empty queue even then the server becomes available for providing service .This case is known as single-vacation system.In this system, the server offers the service in following manner:

(a) The server serves all waiting customers as well as coming customers until all are served and takes vacation on the completion of queue known as exhaustive service policy.

(b) The server provides service to the waiting customers which he finds in the queue after arriving from vacation and will serve to only those waiting customers and then serves remaining customers after arriving from another vacation known as gated service policy.

(c) The server serves only fixed number of customers that are already defined and then proceed for another vacation known as limited service policy.

In the second case, the server avails one more vacation after availing the previous vacation as he finds an empty queue and then comes after the vacation if he finds minimum one customer waiting in the system. This case is known as multiple-vacation system.

Due to various application of vacation system in various fields, a rich review of literature is available. The concept of vacation is first considered by Levy et al [18] in their papers. Doshi [1,2] have also done a tremendous work on vacation models. Takagi [3] and Tian el al [10] also contributed outstandingly on this concept. William J. Gray [14] studied a queueing model with service breakdowns and multiple vacation. He obtained the results for the vacation time and also for the service and repair time following phase type distributions.JihongLi [4] studied the M/M/1 queue with interrupting as well as working vacations and obtained the results by matrix-geometric solution method. Servi et al [6] studied working vacation system where inspite of stop working during vacation , the server offers service at various rate. In this paper, they considered the M/M/1 model and investigated this model with multiple working vacations and obtain the probability generating function(PGF) for the customers of the system. Baba [16] employed the method of matrix analyticity for the study of GI/M/1 model with working vacations. Takine et al [13] studied the time dependent behaviour of M/G/1 queue employed with multiple vacations and gated service discipline. To give the supplementary task in priority queue, Leung [7] has given the scheduling policy for the vacation queueing model. Bymethod of matrix analyticity, Xu et al. [15] derived the PGF for M^X/M/1 queue which has batch arrival and single working vacation. Zhang et al [9] also studied the M/G/1 queue with multiple working as well as interrupting vacations. Baba [17] have also given results for M^X/M/1 queue which has batch arrival and multiple working vacations.Thangaraj [8] developed a queueing system with batch arrival which has concept of two different vacations and two different service patterns. This system provides bulk service only if length of queue exceeds minimum batch size.RashmitaSharma [12] has studied about the bulk queues M^X/G/1 with vacations and state dependent arrival. Gated service vacation was studied by Ramya [5] for queueing models which provides service in two phase.

2. Mathematical Model of the System

In our study we are considering two mathematical models i.e.; D/M/1 queueing model and M/G/1 queueing model. The aim of the study is to establish the analysis of steady state for both these models. Further, we analyzed their steady state behaviour using graphical representation based on given data.

2.1. D/M/1 Model: In this queueing model, D/M/1 queue is a stochastic process where $\{0, 1, 2, 3...\}$ is the state space where these numbers represent the quantity of customers present in the system together with the customer gets serviced. The following hypothesis of the model are

(1) In this model, there is a single queue with a single server.

(2) Arrivals are fixed (deterministic) and service distribution is Markovian which follows exponential distribution.

(3) Let the arrival rate be denoted by λ is constant $(= b)$ sayand the service rate is denoted by μ.

(4) Service discipline is first come first serve basis to serve customers from the front of the queue.

(5) The size of the population can be finite or infinite.

2.2. M/G/1 Model: In this queueing model, M/G/1 queue is a stochastic process where $\{0, 1, 2, 3...\}$ is the state space where these numbers represent the quantity of customers present in the system including the customer gets serviced.

(1) In this model, arrivals are Markovian follows poisson distribution and depar-

tures follow general (arbitrary) distribution.
(2) Let the arrival rate be denoted by λ in which transition of state occurs from state i to $i+1$ i.e.; it represents the arrival of new customer.
(3) The service rate denoted by μ is arbitrary $(= 1/a)$ (say) in which transition of state occurs from state i to $i-1$ i.e.; it represents the departure of existing customer after getting service from the system.
(4) Service time is random and known.
(5) The other assumptions are same as mentioned in (4) and (5) of the above model.

3. State Transition Diagram

State transition diagram is given in the figure 1. In this diagram, when the customers changes from state 0 to 1 and from 1 to 2 and so on, then it is denoted by λ. Further, when the customers changes from state 2 to state 1 and from state 1 to 0 after availing the service, it is denoted by μ. Also when the system is on type I vacation and the system has n customers and it changes to working state i.e;, if state changes from $(n, 1)$ to $(n, 0)$ then the duration of type I vacation is exponentially distributed with mean γ_1. Further when the system is on type II vacation with n customers and changes its state into working state i.e; if state changes from $(n, 2)$ to $(n, 0)$ then the duration of type II vacation is exponentially distributed with mean γ_2.

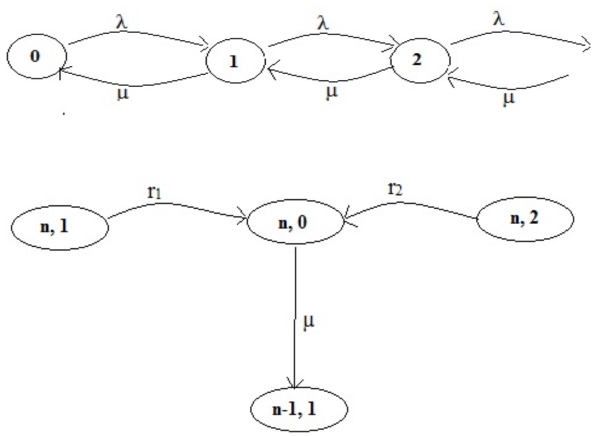

Figure 1

Let $P_{n,k}$ be the probability to be in the state (n, k) for the process. Then, the state transition from one state to another state can be symbolically described as follows:

$$P_{0,1} \xrightarrow{\lambda} P_{1,1} \xrightarrow{\lambda} P_{2,1} \xrightarrow{\lambda} P_{3,1} \xrightarrow{\lambda} \ldots$$

$$P_{0,2} \xrightarrow{\lambda} P_{1,2} \xrightarrow{\lambda} P_{2,2} \xrightarrow{\lambda} P_{3,2} \xrightarrow{\lambda} \ldots$$

$$P_{3,0} \xrightarrow{\mu} P_{2,0} \xrightarrow{\mu} P_{1,0} \xrightarrow{\mu} P_{0,1} \xrightarrow{\mu} \ldots$$

$$P_{1,1} \xrightarrow{\gamma_1} P_{1,0},\ P_{2,1} \xrightarrow{\gamma_1} P_{2,0},\ P_{3,1} \xrightarrow{\gamma_1} P_{3,0} \ldots$$

$$P_{1,2} \xrightarrow{\gamma_1} P_{1,0},\ P_{2,2} \xrightarrow{\gamma_1} P_{2,0},\ P_{3,2} \xrightarrow{\gamma_1} P_{3,0} \ldots$$

4. Assumptions for Steady State Analysis

Here we are considering a queueing system which consists of multiple vacations in which arrival rate is denoted by λ and service rate by μ. Note that here in the vacation system $\mu > \lambda$. In our study we assume two types of vacations. Type I vacation is taken by the server when all the waiting customers are being served i.e; server takes vacation after busy period. Type II vacation is taken by the server when he comes back after first vacation and finds an empty queue and so he takes an another vacation. Type I and type II vacations are exponentially distributed with a mean of $1/\gamma_1$ and $1/\gamma_2$ respectively.

Let the system state be denoted by (n, k) where $n =$ number of customers present in the system, $k = 0$ means server is in working state, $k = 1$ means server is on type I vacation, $k = 2$ means server is on type II vacation.

Suppose the probability to be in the state (n, k) at time t is given by $P_{n,k}(t)$. Then

$$P_{n,k} = \lim_{t\to\infty} P_{n,k}(t)$$

Consider the result given by Oliver C. Ibe and Olubukola A. Isijola [11] on M/M/1 queueing model.

The steady state probability $P_{n,k)}$ of M/M/1 model is given by

$$P_{n,k} = \begin{cases} \rho\left[\frac{\alpha_1\beta_1(\beta_1^{n-1}-\rho^{n-1})}{\beta_1-\rho} + \frac{\alpha_2\beta_2(\beta_2^{n-1}-\rho^{n-1})}{\beta_2-\rho} + \rho^{n-2}\right] p_{1,0}, & k = 0 \\ \alpha_1\beta_1^n p_{1,0}, & k = 1 \\ \alpha_2\beta_2^n p_{1,0}, & k = 2 \end{cases} \tag{1}$$

where

$$p_{1,0} = ((1-\rho)(1-\beta_1)(1-\beta_2)) \times ((1-\beta_1)(1-\beta_2) + \alpha_1(1-\beta_1) \times \{1-\rho(1-\beta_1)\}$$
$$+\alpha_2(1-\beta_1) \times \{1-\rho(1-\beta_2)\})^{-1}$$

Here $\rho = \lambda/\mu$ denotes offered load,

$$\alpha_1 = \frac{\mu}{\lambda+\gamma_1},\quad \alpha_2 = \frac{\mu\gamma_1}{\lambda(\lambda+\gamma_1)},\quad \beta_1 = \frac{\lambda}{\lambda+\gamma_1} < 1,\quad \beta_2 = \frac{\lambda}{\lambda+\gamma_2} < 1$$

Using the result from equation (1), given by Oliver C. Ibe and Olubukola A. Isijola [11], we establish the steady state probability $P_{n,k}$ for D/M/1 and M/G/1 queueing models.

5. Steady State Analysis of D/M/1 Model and M/G/1 Model

Considering the analysis of steady state of M/M/1 multiple vacation queueing model [11] given by equation (1), we derive and analyze the steady state of D/M/1 Model and M/G/1 Model.

5.1. Steady State Analysis of D/M/1 Model

In D/M/1 model, arrivals are fixed and departures are exponential. Thus, the factor of utilization is given by

$$\rho = \frac{\lambda}{\mu} = \frac{b}{\mu} \quad \text{as} \quad \lambda = \text{constant} \quad = b(\text{say}) \tag{2}$$

Using equation (2) in equation (1) and after simplifying, we get steady state probability for D/M/1 queueing model given by

$$P_{n,k} = \begin{cases} \frac{b}{\mu}\left[\frac{\alpha_1\beta_1(\beta_1^{n-1}-\left(\frac{b}{\mu}\right)^{n-1})}{\beta_1-\left(\frac{b}{\mu}\right)} + \frac{\alpha_2\beta_2(\beta_2^{n-1}-\left(\frac{b}{\mu}\right)^{n-1})}{\beta_2-\left(\frac{b}{\mu}\right)} + \left(\frac{b}{\mu}\right)^{n-2}\right] p_{1,0}, & k = 0 \\ \alpha_1\beta_1^n p_{1,0}, & k = 1 \\ \alpha_2\beta_2^n p_{1,0}, & k = 2 \end{cases} \tag{3}$$

where

$$P_{1,0} = \frac{\left(1-\frac{b}{\mu}\right)(1-\beta_1)(1-\beta_2)}{\alpha_1(1-\beta_2)\left(1-\frac{b}{\mu}\right) + \alpha_2\left(1-\frac{b}{\mu}\right)(1-\beta_1) + \alpha_1\beta_1\left(\frac{b}{\mu}\right)(1-\beta_2) + \alpha_2\beta_2}$$

$$\frac{1}{\left(\frac{b}{\mu}\right)(1-\beta_1) + (1-\beta_1)(1-\beta_2)} \tag{4}$$

5.2. Steady State Analysis of M/G/1 Model

In M/G/1 Model, arrivals have poisson distribution and departures are arbitrary. Thus, the factor of utilization is given by

$$\rho = \frac{\lambda}{\mu} = \frac{\lambda}{1/a} = a\lambda \quad \text{as} \quad \mu = =1/\text{a} \quad (\text{say}) \tag{5}$$

Using equation (5) in equation (1) and after simplifying, we get steady state probability for M/G/1 queueing model given by

$$P_{n,k} = \begin{cases} (a\lambda)\left[\frac{\alpha_1\beta_1(\beta_1^{n-1}-(a\lambda)^{n-1})}{\beta_1-(a\lambda)} + \frac{\alpha_2\beta_2(\beta_2^{n-1}-(a\lambda)^{n-1})}{\beta_2-(a\lambda)} + (a\lambda)^{n-2}\right] p_{1,0}, & k=0 \\ \alpha_1\beta_1^n p_{1,0}, & k=1 \\ \alpha_2\beta_2^n p_{1,0}, & k=2 \end{cases} \tag{6}$$

where

$$P_{1,0} = \frac{(1-a\lambda)\,(1-\beta_1)(1-\beta_2)}{\alpha_1(1-\beta_2)\,(1-a\lambda) + \alpha_2\,(1-a\lambda)\,(1-\beta_1) + \alpha_1\beta_1\,(a\lambda)\,(1-\beta_2) + \alpha_2\beta_2}$$

$$\frac{1}{(a\lambda)\,(1-\beta_1) + (1-\beta_1)(1-\beta_2)} \tag{7}$$

Hence, equations (3) and (6) gives the probability for steady state for D/M/1 and M/G/1 queueing models respectively.

6. Calculations for Steady State Analysis of D/M/1 Model and M/G/1 Model

The data used in this paper for analysis is based on observation of any real world situation. For study of steady state of D/M/1 model, we have done calculations using the equations (3) and (4) and for M/G/1 model using the equations (6) and (7). Consider a four wheeler service centre that has certain service policies. According to their policy, if there is no customer for servicing of four wheeler, the server go to the vacation and start performing other work of service centre. When server comes back from vacation and founds no customer then again he takes another break. Further if he founds even a single customer, he starts servicing the customer and will not take break until that customer gets serviced. The server utilizes the break in some other activities of service centre or may be for personal reason such as coffee break. The break taken by server can be short or can be of long duration. There may be break due to some technical problems in the system. These problems may be temporary or permanent. There are following observations for the arrival and departure of the four wheelers in the service centre. The entire day of the service centre from 9:00 A.M. to 9:00 P.M. is categorized into six time intervals. Further note that, in D/M/1 Model arrivals are constant per unit of time and in M/G/1 Model departures are arbitrary.

Also we have assumed that total no. of customers present in the system are $n = 6$, mean duration of type I vacation $\gamma_1 = 10$ minutes, mean duration of type II vacation $\gamma_2 = 12$ minutes.

Table I
Data for D/M/1 Model

S.No.	Time Interval	Arrival Rate of Vehicles (λ=constant)	Service Rate of Vehicles(μ)	Utilization Factor ($\rho = \frac{\lambda}{\mu}$)
1	9:00 A.M.- 11:00 A.M.	5	8	0.625
2	11:00 A.M. - 1:00 P.M.	5	6	0.833
3	1:00 P.M.- 3:00 P.M.	5	9	0.555
4	3:00 P.M.- 5:00 P.M.	5	8	0.625
5	5:00 P.M.- 7:00 P.M.	5	7	0.714
6	7:00 P.M. – 9:00 P.M.	5	7	0.714

Table II

Analysis of Steady State for D/M/1 Model

S.No.	Time Interval	α_1	α_2	β_1	β_2	Steady State when server is on type I vacation $(P_{n,k})$ at $k = 1$	Steady State when server is on type II vacation $(P_{n,k})$ at $k = 2$	Steady State when server is in working state $(P_{n,k})$ at $k = 0$
1	9:00 A.M.- 11:00 A.M.	0.444	1.066	0.333	0.294	0.0001017	0.0001156	0.03013905006
2	11:00 A.M. - 1:00 P.M.	0.375	0.8	0.333	0.294	0.0000497	0.00005025	0.06115496199
3	1:00 P.M.- 3:00 P.M.	0.473	1.2	0.333	0.294	0.0001157	0.0001390	0.01966852601
4	3:00 P.M.- 5:00 P.M.	0.444	1.066	0.333	0.294	0.0001017	0.0001156	0.03013905006
5	5:00v P.M.- 7:00 P.M.	0.411	0.28	0.333	0.294	0.0001054	0.0000340	0.00002716813
6	7:00 P.M. – 9:00 P.M.	0.411	0.28	0.333	0.294	0.0001054	0.0000340	0.00002716813

Table III
Data for M/G/1 Model

S.No.	Time Interval	Arrival Rate of Vehicles (λ)	Service Rate of Vehicles(μ=1/a)(arbitrary)	Utilization Factor ($\rho = \frac{\lambda}{\mu}$)
1	9:00 A.M.- 11:00 A.M.	3	5	0.714
2	11:00 A.M. - 1:00	3	6	0.6

	P.M.			
3	1:00 P.M.- 3:00 P.M.	5	7	0.85
4	3:00 P.M.- 5:00 P.M.	6	7	0.714
5	5:00 P.M.- 7:00 P.M.	5	8	0.625
6	7:00 P.M. – 9:00 P.M.	5	8	0.625

Table IV

Analysis of Steady State for M/G/1 Model

S. No.	Time Interval	α_1	α_2	β_1	β_2	Steady State when server is on type I vacation $(P_{n,k})$ at $k = 1$	Steady State when server is on type II vacation $(P_{n,k})$ at $k = 2$	Steady State when server is in working state $(P_{n,k})$ at $k = 0$
1	9:00 A.M.- 11:00 A.M.	0.333	1.28	0.230	0.2	0.000009462	0.00001586	0.0226773912
2	11:00 A.M. - 1:00 P.M.	0.375	1.538	0.2307	0.2	0.00001152148	0.00002005986	0.01097003489
3	1:00 P.M.- 3:00 P.M.	0.411	0.28	0.333	0.294	0.0001054	0.000034032	0.0000271681
4	3:00 P.M.- 5:00 P.M.	0.411	0.7291	0.375	0.333	0.000096120	0.00008360	0.06465987599
5	5:00 P.M.- 7:00 P.M.	0.444	1.066	0.33	0.2941	0.00009646617	0.0001160475	0.03010111826
6	7:00 P.M. – 9:00 P.M.	0.444	1.066	0.33	0.2941	0.00009646617	0.0001160475	0.03010111826

7. Graphical Representation Using MATLAB

The analysis of steady state for D/M/1 and M/G/1 model in various states is shown below through the graphical representation using the software MATLAB. Graphs 7.1 and 7.2 shows the analysis of steady state of D/M/1 queueing model in type I and type II vacations. The analysis shows that D/M/1 queueing model attains the steady state after some time in all cases. Further graphs 7.3 and 7.4 shows the analysis of steady state of M/G/1 queueing model in type I and type II vacations. Here, also the analysis shows that M/G/1 queueing model also attains the steady state after some time in all cases. Graphs 7.5 and 7.6 shows the analysis of steady state in D/M/1 Model and M/G/1 Models respectively in working state. Graph 7.7 shows the comparative study of analysis of steady state of Type I vacation in D/M/1 and M/G/1 Models. The study shows that in case of type I vacations, the steady state is obtained at lower values in M/G/1 Model than D/M/1 Model. Graph 7.8 shows the comparative study of analysis of steady state of Type II vacation in D/M/1 and M/G/1 Models. The study shows that in case of type II vacations, the steady state is obtained at lower values in D/M/1 Model than M/G/1 Model.Graph 7.9 shows the comparative study of analysis of steady state of working state in D/M/1 and M/G/1 Models. The study shows that in case of working state, the steady state is obtained at lower values in M/G/1 Model than D/M/1 Model.

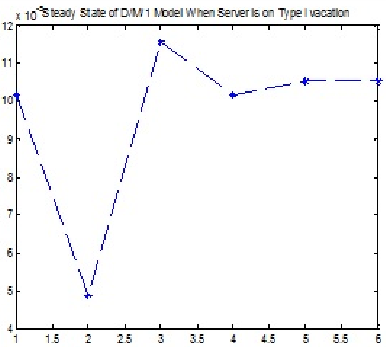

Fig 7.1

X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

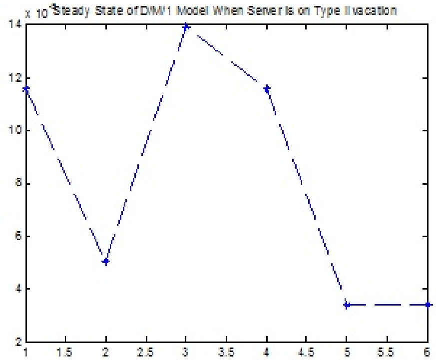

Fig 7.2
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

Fig. 7.1 is the analysis of steady state of D/M/1 Model when the server is on type I vacation and Fig. 7.2 is the analysis of steady state of D/M/1 Model when the server is on type II vacation. The analysis shows that in comparison to type I vacation, steady state in case of type II vacation is obtained at lower values in case of D/M/1 Model. Thus, the study suggests that type II vacation is better in comparison to type I vacation in D/M/1 Model.

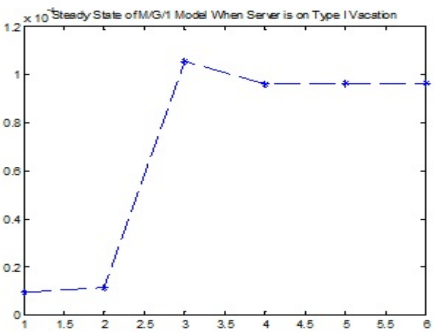

Fig 7.3
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

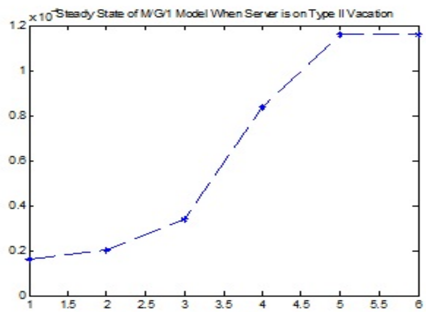

Fig 7.4

X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

Fig. 7.3 is the analysis of steady state of M/G/1 Model when the server is on type I vacation and Fig. 7.4 is the analysis of steady state of M/G/1 Model when the server is on type II vacation. The analysis shows that in comparison to type II vacation, steady state in case of type I vacation is obtained at lower values in case of M/G/1 Model. Thus, the study suggests that type I vacation is better in comparison to type II vacation in M/G/1 Model.

Fig 7.5
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

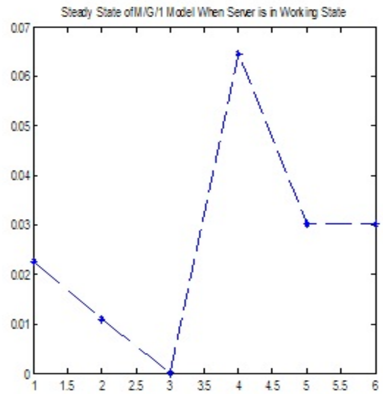

Fig 7.6
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

Fig. 7.5 is the analysis of steady state of D/M/1 Model when the server is in the working state and Fig. 7.6 is the analysis of steady state of M/G/1 Model when the server is in the working state. The analysis shows that in comparison to working state of M/G/1 Model, steady state in case of D/M/1 Model is obtained at lower values in working state.

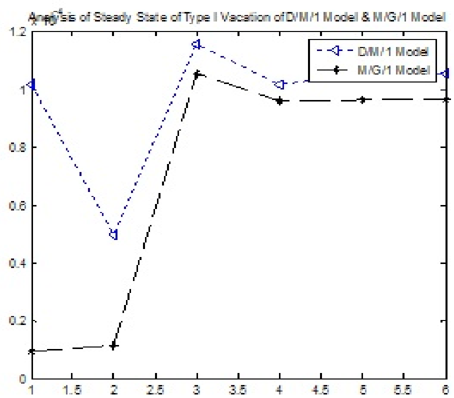

Fig 7.7
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

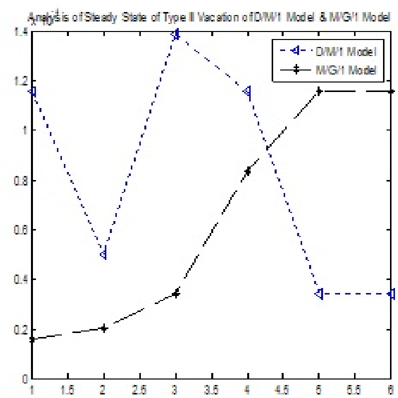

Fig 7.8
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

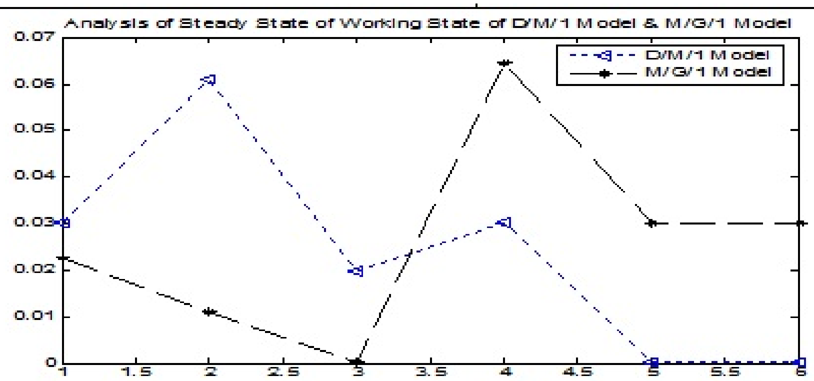

Fig 7.9
X-axis : Time Interval
Y-axis : Steady State ($P_{n,k}$)

Fig 7.7 to 7.9 is the comparative analysis of steady state in D/M/1 and M/G/1 Models in type I, type II vacations and in working state respectively.

8. Conclusion

In this paper, we deal with the queueing system having multiple vacations in which there are two types of vacation. In type I vacation the server goes to the vacation when all the customers waiting in the queue get served. In type II vacation, the server avails one more break if he still finds an empty queue after availing the first break. In this paper, we have examined the behaviour of system over time known as steady state. For our study we have considered two queueing models i.e; D/M/1 Model and M/G/1 Model and then we found the steady state solution of both these models when the server is in working state and when the server is on type I and type II vacation. Further to analyze the steady state of the system when server is in working state and on type I and type II vacation, we have taken a problem of four wheeler service centre from the real world where the entire day is categorized into six time intervals. In our analysis we found that as the time increases our system obtains the steady state i.e.; the system becomes independent of time. The graphical representation of analysis of steady state when server is in various states is also performed using the software MATLAB.This study can be used in various real life experiences where vacations are applicable.

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